6.1 Alternating current

F-X
Ext

Let's begin Patterns formed by light emitting diodes

1 A.c. and d.c.

Check-point 1

2 Effective value of an a.c.

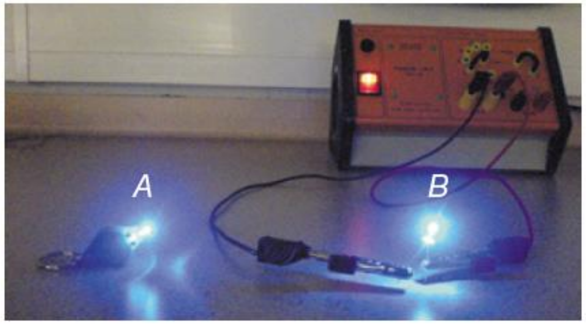
Root-mean-square value of an a.c.

3 Check-point 2

4 Root-mean-square and peak values

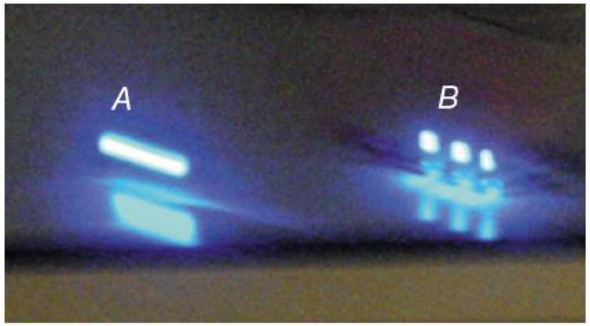
Check-point 3

Patterns formed by light emitting diodes



A and *B* are identical light emitting diodes (LEDs).

A is connected to a battery, while *B* an a.c. power supply.



Take a photo of them with a moving camera:

A $\Rightarrow$ line pattern   *B* $\Rightarrow$ dot pattern

Why are different patterns formed by LEDs?

1 A.c. and d.c.

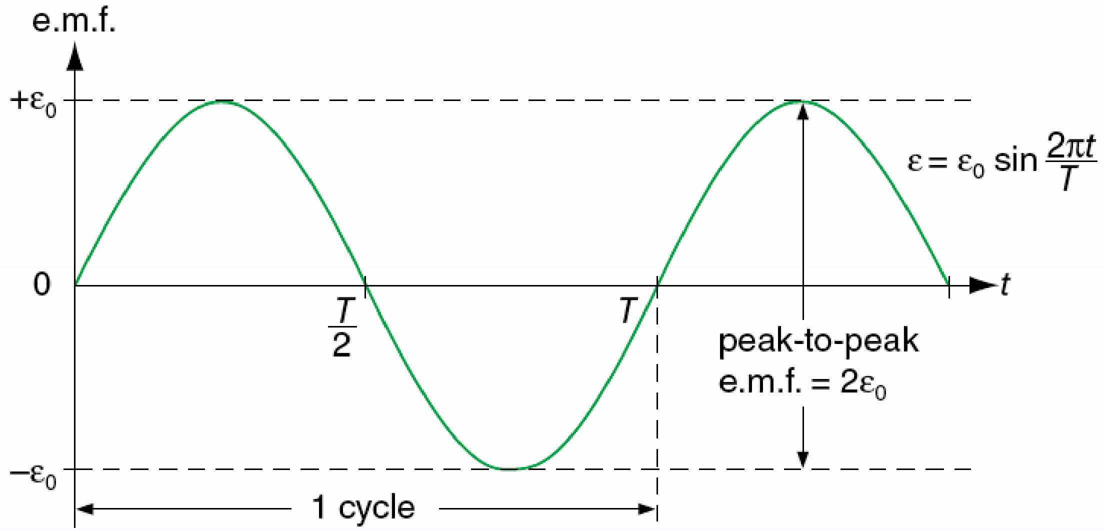
Expt 6a

Current and e.m.f. produced by a small a.c. generator (dynamo)

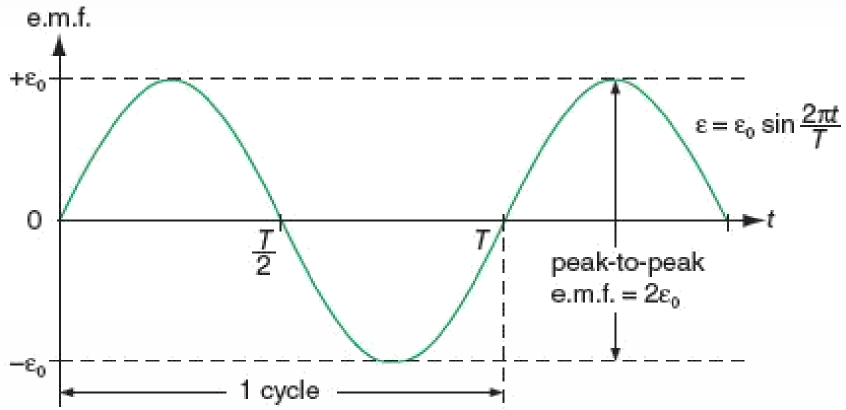
1 A.c. and d.c.

From Expt 6a: the current and the e.m.f. produced by a simple a.c. dynamo change direction periodically.

An a.c. varies periodically with time in both magnitude and direction.



1 A.c. and d.c.



Cycle: 1 complete alternation

Frequency: number of cycles per second

Period: duration of one cycle

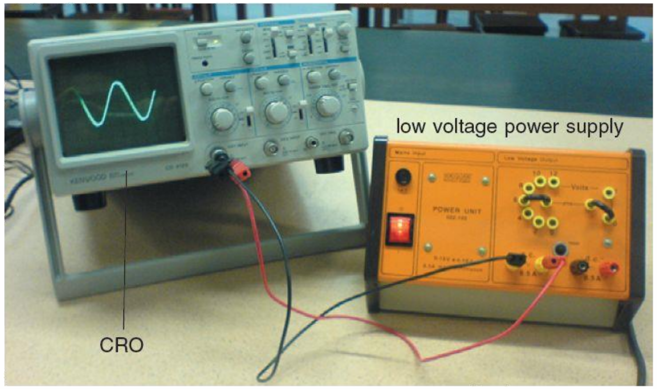
Peak value of e.m.f. = ε_0

$\Rightarrow$ peak-to-peak value of e.m.f. = $2\varepsilon_0$

1 A.c. and d.c.

a Sinusoidal a.c.

The CRO shows a sinusoidal alternating voltage output from a low voltage power supply operated with the a.c. mains.



Period = 0.02 s,

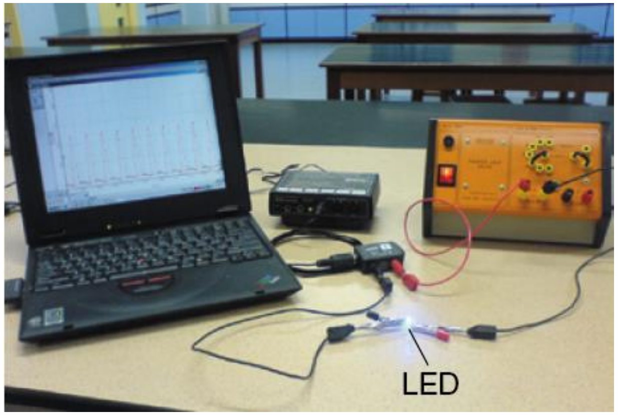
i.e. frequency = 50 Hz

= frequency of electricity supply in HK

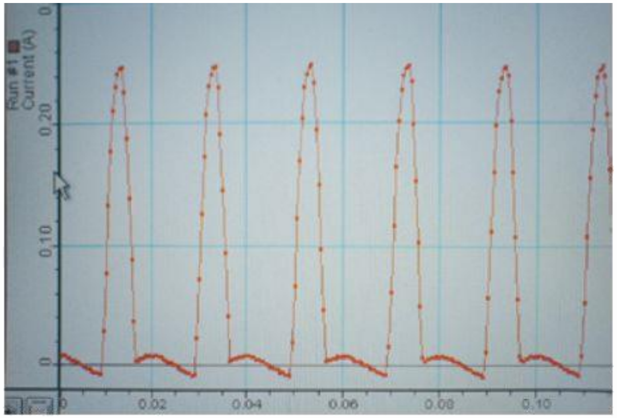
1 A.c. and d.c.

b Varying d.c.

When a.c. flows through a diode, half cycles instead of full cycles are left (∵ diode permits current to flow only in one direction).



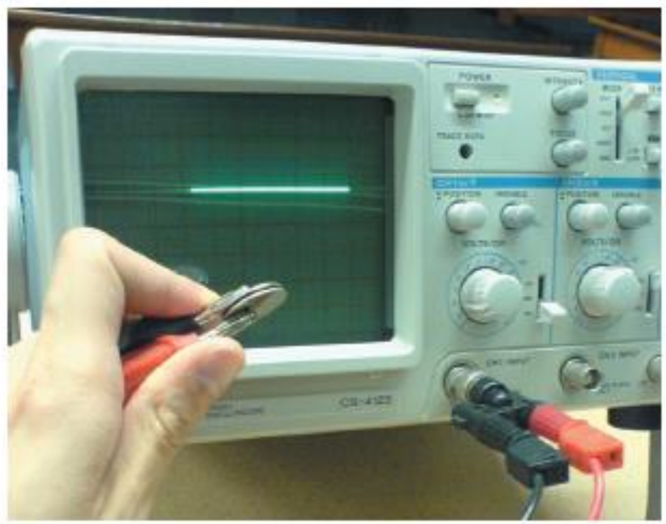
When an LED is connected to a.c. power supply with current sensor, the sensor measures a varying d.c.



1 A.c. and d.c.

c Constant d.c.

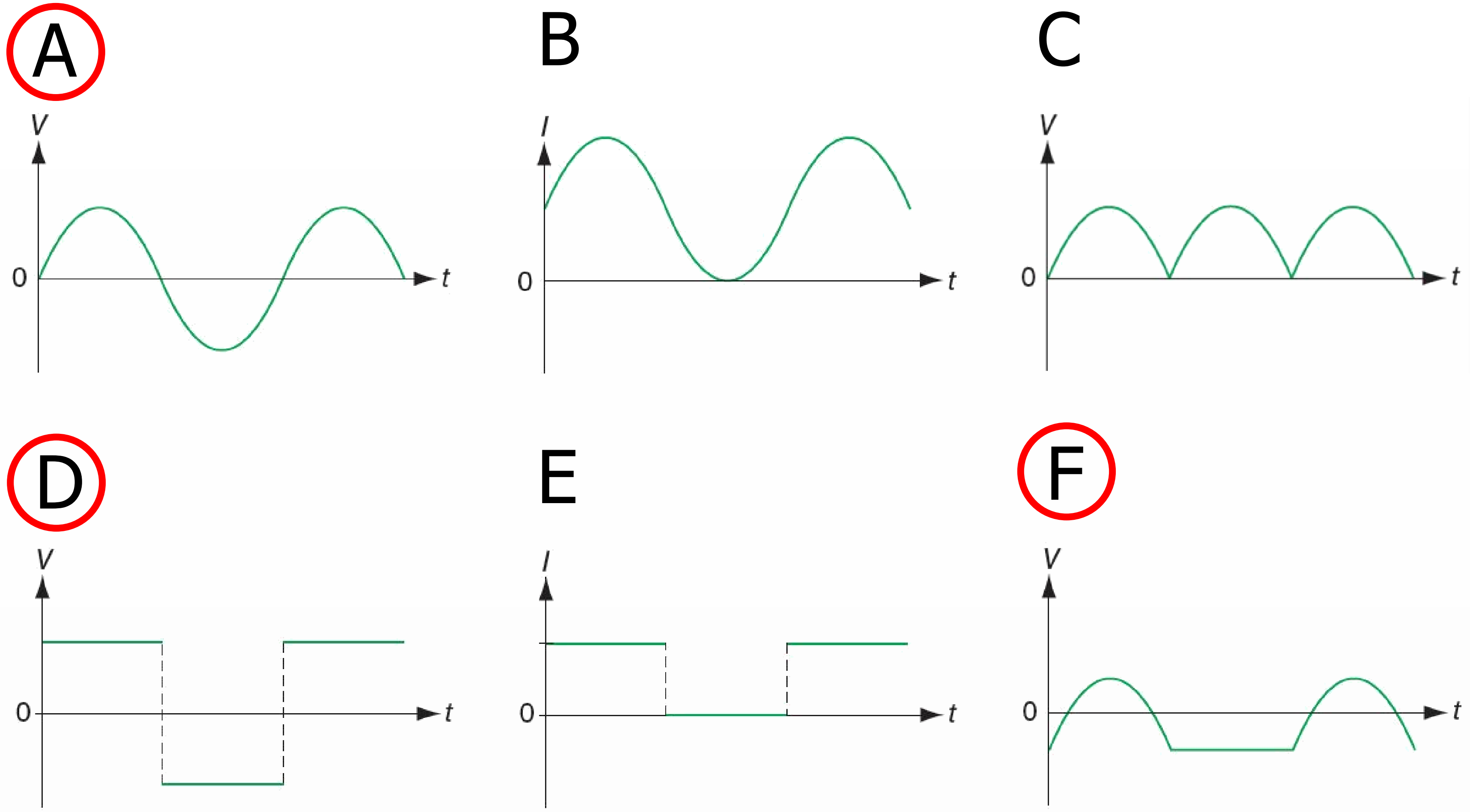
The type of d.c. that we commonly use is a steady or constant d.c., which gives a horizontal line on a CRO.



Example 1 **Different patterns formed by LEDs**

Check-point 1 – Q1

Which are a.c. currents/voltages?



2 Effective value of an a.c.

Expt 6b

The effective values of alternating current and voltage

2 Effective value of an a.c.

From Expt 6b:

Same brightness

$\Rightarrow$ Equal average power developed by the a.c. and the chosen d.c.

$\Rightarrow$ size of steady d.c. = **effective value** of a.c.

> The effective value of an a.c. is equal to the steady current which produces the same heating effect in the same resistance in the same time.

3 Root-mean-square value of an a.c.

When a light bulb of resistance R is lit by an a.c. I, instantaneous power of the bulb

$P = I^2 R$ ....(1)

Mean or the average power

$\overline{P} = \overline{I^2 R} = \overline{I^2} R$ ....(2)

A steady current I_{dc} gives the same heating effect (in terms of brightness) in light bulb.

Power of the bulb $P_{dc} = I_{dc}^2 R$ ....(3)

3 Root-mean-square value of an a.c.

A.c & d.c circuits give the same heating effect:

$$\overline{I^2 R} = I_{dc}^2 R$$

$$\Rightarrow I_{dc} = \sqrt{\overline{I^2}} \quad \ldots\ldots\ldots\ldots\ldots (4)$$

i.e. steady current I_{dc} of value $\sqrt{\overline{I^2}}$ produces the same heating effect as the a.c. I in bulb.

$\therefore \sqrt{\overline{I^2}}$ represents the effective value of the a.c.

3 Root-mean-square value of an a.c.

$\sqrt{\overline{I^2}}$ can be obtained by taking square root of the mean/average value of the square of current.

$\Rightarrow$ **root-mean-square value** (r.m.s. value) of the current, denoted by I_{rms}.

The effective value of an a.c. is the r.m.s. value of the current.

$$I_{\text{rms}} = \sqrt{\overline{I^2}} = \sqrt{\text{mean value of } I^2}$$

3 Root-mean-square value of an a.c.

Similarly, effective value of the alternating voltage is the r.m.s. value of the voltage, denoted by V_{rms}.

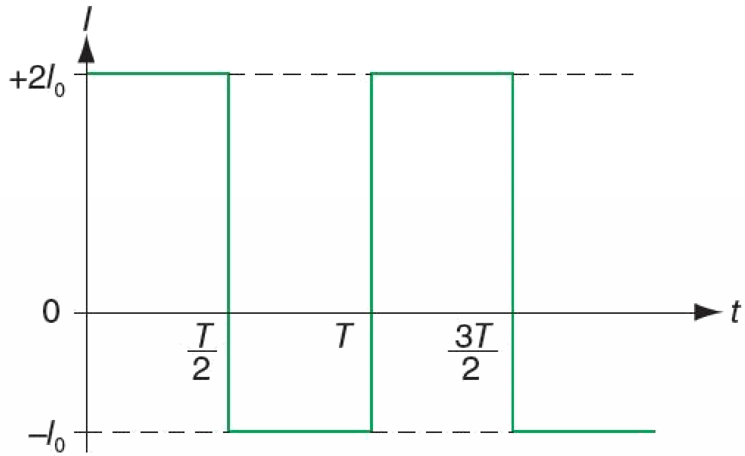
$$\overline{P} = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R} = V_{\text{rms}} I_{\text{rms}} \quad \text{..............(5)}$$

Example 2 R.m.s. value of an alternating current

Check-point 2 – Q1

The current of an a.c. source varies with time.

(a) Express I_{rms} in terms of I_0.



$$I_{rms} = \sqrt{\frac{(2I_0)^2 + (-I_0)^2}{2}}$$

$$= \sqrt{\frac{5}{2}} I_0$$

(b) If 0.5 A of d.c. gives the same heating effect as this a.c., value of I_0 = ?

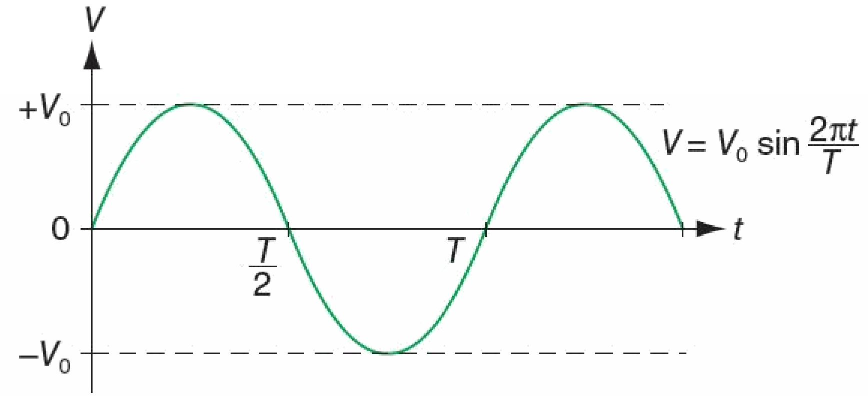
$$\sqrt{\frac{5}{2}} I_0 = 0.5 \Rightarrow I_0 = 0.316 \text{ A}$$

4 Root-mean-square and peak values

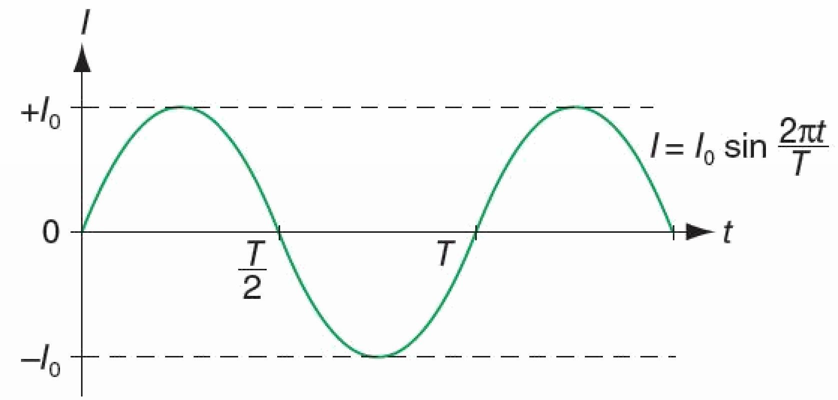
The relationship between V_{rms}, I_{rms} and peak values V_0, I_0 depends on the waveform of a.c.

For a sinusoidal case,

$$V = V_0 \sin \frac{2\pi t}{T} \quad \text{........(6)}$$



$$I = I_0 \sin \frac{2\pi t}{T} \quad \text{........(7)}$$



4 Root-mean-square and peak values

From (1) and (7), the instantaneous power developed by a sinusoidal a.c. in a pure resistor is

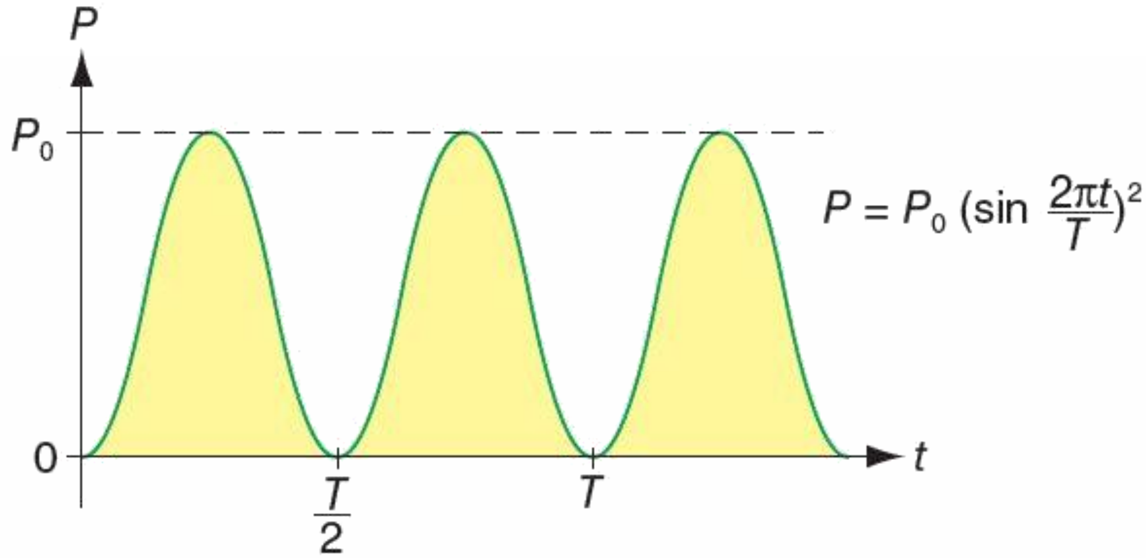
$$P = I^2 R = \left(I_0 \sin \frac{2\pi t}{T}\right)^2 R$$

$$= I_0^2 R \left(\sin \frac{2\pi t}{T}\right)^2$$

$$P = P_0 \left(\sin \frac{2\pi t}{T}\right)^2 \text{.....................(8)}$$

4 Root-mean-square and peak values

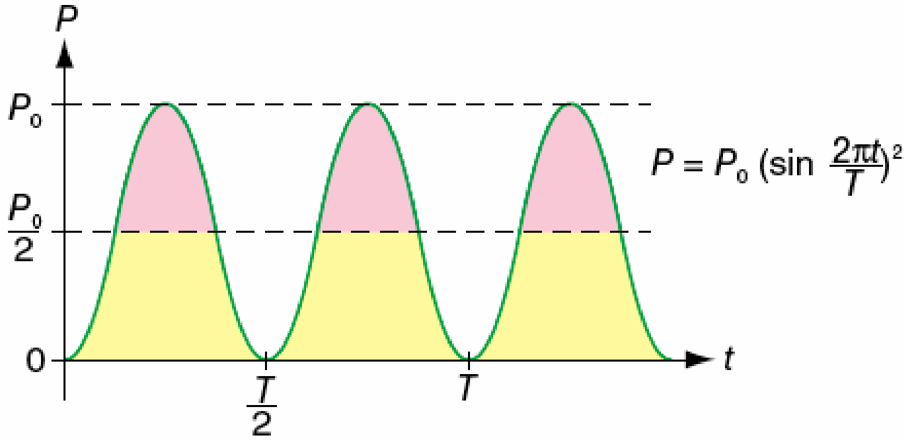
How the power P for a pure resistor varies with time t :

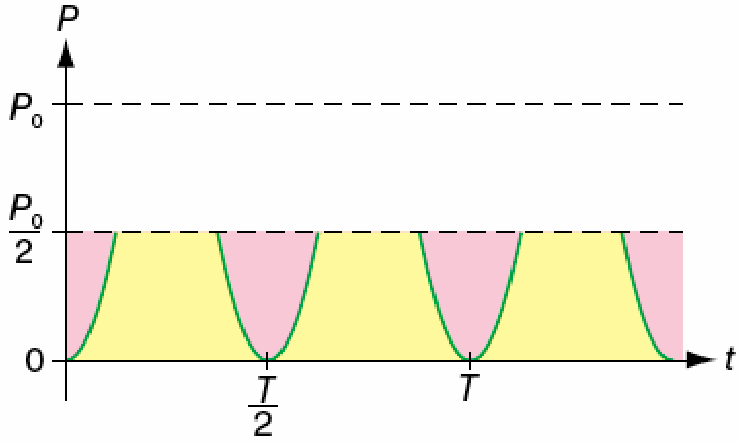


The value of power is non-negative and fluctuating between 0 and max. power P_0.

4 Root-mean-square and peak values

Areas above the dotted line (passing through $\frac{1}{2}P_0$) fit into the empty regions over completed cycles.





In this case, the mean power $\overline{P} = \frac{1}{2}P_0$...............(9)

4 Root-mean-square and peak values

Using (5), (9) and the max. power $P_0 = I_0^2 R = \frac{V_0^2}{R}$,

$$\overline{P} = \frac{1}{2} P_0 \Rightarrow I_{rms}^2 R = \frac{1}{2} I_0^2 R \quad \& \quad \frac{V_{rms}^2}{R} = \frac{1}{2}\left(\frac{V_0^2}{R}\right)$$

$$\Rightarrow \boxed{I_{rms} = \frac{I_0}{\sqrt{2}}} \quad \& \quad \boxed{V_{rms} = \frac{V_0}{\sqrt{2}}} \text{ (for sinusoidal a.c.)}$$

The voltage of the mains in Hong Kong, 220 V, is the r.m.s. value.

Example 3 Peak value of mains supply in Hong Kong

Check-point 3 – Q1

A sinusoidal a.c. of 60 Hz has a peak value of 15 A.

Find the r.m.s. value of the a.c.

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$$

$$= \frac{15}{\sqrt{2}}$$

$$= 10.6 \text{ A}$$

Check-point 3 – Q2

Find the max. voltage from an a.c. mains of 110 V.

Max. voltage $= V_{rms}\sqrt{2}$

$= 110\sqrt{2}$

$= 156$ V

The End